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THE SYNTHESIS OF MINIMUM
SENSITIVITY NETWORKS

By

Jomes D. Schoeffler

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A.D.Waren

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Technical Report Number Six

Engineering Division

Case Institute of Technology

Cleveland 6, Ohio

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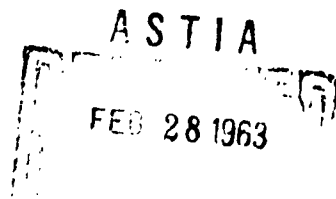
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ABSTRACT

The synthesis of networks with minimum sensitivity to element tolerances is studied from a computer viewpoint. The theory of equivalent networks is used to generate a sequence of networks whose transfer functions are identical to that of a given network but whose elements differ from one network to the next by an incremental amount. In the limit, differential equations result whose solution at any value of the independent variable give the elements of an equivalent network. Similarly, differential equations for the sensitivity of the transfer function to changes in each of the elements are derived. The differential equations in both cases are linear homogeneous with the elements of the transformation matrix as the forcing functions. With the aid of the exponential solution to the matrix differential equation, digital computer solution even for complex networks is very straightforward and rapid. As a measure of the optimality of the network, the sum of the magnitudes of the sensitivities is chosen as a performance criterion. The method of steepest descent applied to this criterion leads to a simple choice of the transformation parameters which is easily implemented on the digital computer, thereby allowing efficient synthesis of networks with minimum sensitivity to element tolerances.

The Synthesis of Minimum Sensitivity Networks

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1.   Introduction:

The advent of thin film and integrated circuit techniques
has changed some of the criteria by which networks are eval-
uated. In particular, the new techniques place less emphasis
on the number of elements in a network but require designs
which are fairly insensitive to changes in the element values,
due to the difficulty of maintaining tolerances at this stage
in the development of the thin film techniques. Several authors
have recently considered the design of insensitive networks[1-4]
from various viewpoints. In this paper, it is shown that the
theory of equivalent networks is very useful in the synthesis
of networks insensitive to large element tolerances.

Cauer in 1929 showed that by means of a congruence trans-
formation, one physically realizable network could be generated
from another in such a way that specified driving point and/or
transfer functions were held invariant.[5,6] This approach to
network synthesis has been discussed by many authors and with
the exception of the minimum-inductance transformation in fil-
ter theory has not realized its apparent potentialities.[7-10]
Useful results from this theory can be derived by using the
concept of continuously equivalent network theory together with

-1-

a digital computer for implementation.[11] In this paper, this approach is used.

Given a network with the desired transfer function (synthesized by any of the known schemes), the theory of equivalent networks is used to generate another network with the same transfer function but with elements differing from those of the original network by an incremental amount. In the limit, differential equations result whose solution at any value of the independent variable give the elements of an equivalent network. In this way, it is possible to generate many networks equivalent to a given network but having widely differing element values while insuring at all times that no elements become negative. In addition, differential equations for the sensitivities of the transfer function to changes in element values are derived and an efficient computational algorithm derived which allows rapid realization of minimal sensitive networks on a computer. In an example 30 element network, computing times of several minutes were found.

2. Equivalent Network Theory:

Consider a network with n independent node-pairs. Such a network is described by a set of n equations of the form

$$I = Y_o E \qquad 2.1$$

where Y_o is the nxn admittance matrix of the network, E is
the nxl column matrix of node voltages and I is the nxl column
matrix of source currents. If this network is imbedded in a
2n-port network of ideal transformers, there results another
n-port network and the variables of the original and new net-
work are related by a linear transformation:

$$E \quad = AE'$$

$$I' \quad = A^t I \qquad\qquad 2.2$$

where A is an nxn nonsingular constant matrix, t denotes trans-
pose, and E' and I' are the new voltage and current variables.[9]
The last equation follows from the losslessness of the trans-
former network. The new variables are related by the equation

$$I' = Y'E' \qquad\qquad 2.3$$

where

$$Y' = A^t Y_o A \qquad\qquad 2.4$$

From the method of construction it is clear that the resulting
admittance matrix is physically realizable. The usefulness
stems from the possibility of interpreting Eq. 2.3 as the equi-
librium equations of an equivalent transformerless network.
In this case, not all elements will be positive in general and
this fact has limited the applications of the theory.[10] By
proper choice of the transformation matrix A, certain driving
point and/or transfer functions can be held invariant. For

example, if the kth row of A is the kth unit vector (all zeros except for the kth entry which is unity), the driving point impedance at the kth terminal pair is invariant to the transformation. If two rows are so chosen, the driving point impedance at each port is invariant and also the transfer impedance between the two ports. Thus it is possible to maintain desired transfer functions invariant to the transformation.[9]

The theory becomes more useful if we imagine a transformation from a given network to one whose elements differ by only a small amount, and then pass to the limit. Let

$$A = U + B \Delta x \qquad 2.5$$

$$|b_{ij}| \leq 1$$

where the elements of B are bounded, and x in an independent variable. Consider the admittance matrix of the given network as a function of the independent variable x, Y(x) so that the given admittance matrix is $Y(0) = Y_0$. Then applying this transformation yields a new matrix $Y(x + \Delta x)$ given by

$$Y(x + \Delta x) = Y(x) + [B^t Y(x) + Y(x)B]\Delta x + B^t Y(x)B(\Delta x)^2$$

$$2.6$$

In the limit as Δx approaches zero, there results the matrix

differential equation

$$\frac{dY}{dx} = B^t Y + YB$$

with initial conditions

$$Y(0) = Y_o \qquad\qquad 2.7$$

For any choice of $B(x)$, the solution to this set of differ-
ential equations at any value of x gives a realizable admit-
tance matrix. In order to maintain some transfer functions
invariant, note from Eq. 2.5 that any row of A which is a
unit vector implies that the same row of B must be identically
zero. That is, equivalence at a port is assured if the row
of B corresponding to that port is zero. The problem of nega-
tive elements which plagues the usual formulation of the
equivalent network theory problem is easily surmounted here
because the elements of the admittance matrix change contin-
uously as a function of the independent variable x and the
transformation matrix B is also chosen continuously as a func-
tion of x, thereby allowing choices for B which keep elements
positive or zero.

The set of equations 2.7 really consist of $3n^2$ equations
since the nxn matrices contain n^2 entries and the admittance
matrix is really the sum of a conductance matrix, a capacitance
matrix, and an inductance matrix. Since the node-to-datum set
of equations for an RLC network have symmetrical admittance

matrices, only $3(n^2 + n)/2$ of the equations are different. The equations can be put into a more useful form for study and numerical solution by considering the relation between the admittance matrix and the elements of the network. The entry in the ith row and j^{th} column ($i \neq j$) of the admittance matrix of a node to datum set of variables is the negative of the admittance connected between the ith and j^{th} nodes, and each entry on the diagonal is merely the sum of all admittance attached to the corresponding node. Hence it is possible to derive the elements of the network from the admittance matrix (this assumes no mutual coupling between elements of the network).

Hence it is possible to convert the differential equations into another set whose variables are the elements of the network. For example, consider the RC network in Fig. 1 having two independent mode-pairs. The matrix differential equation is

$$\frac{dY}{dx} = B^t Y + YB \qquad 2.8$$

where

$$Y = \begin{bmatrix} g_1 + g_2 & - g_1 \\ - g_1 & g_1 + g_3 \end{bmatrix} + S \begin{bmatrix} c_1 + c_2 & - c_1 \\ - c_1 & c_1 + c_3 \end{bmatrix}$$

and

$$B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \qquad 2.10$$

This is the set of equations

$$g_1' + g_2' = 2(b_1 g_1 + b_1 g_2 - b_3 g_1)$$

$$- g_1' = - b_1 g_1 + b_3 g_1 + b_3 g_3 + b_2 g_1 + b_2 g_2 - b_4 g_1$$

$$g_1' + g_3' = 2(b_4 g_1 + b_4 g_3 - b_2 g_1)$$

$$c_1' + c_2' = 2(b_1 c_1 + b_1 c_2 - b_3 c_1)$$

$$- c_1' = - b_1 c_1 + b_3 c_1 + b_3 c_3 + b_2 c_1 + b_2 c_2 - b_4 c_1$$

$$c_1' + c_3' = 2(b_4 c_1 + b_4 c_3 - b_2 c_1) \qquad 2.11$$

Here the prime denotes differentiation with respect to x and it is important to note that each b_i is in general an arbitrary (but bounded) function of x.

Note that this set can be put into the convenient form of a first order matrix differential equation:

$$\frac{dG}{dx} = M_1 G$$

$$\frac{dC}{dx} = M_1 C \qquad 2.12$$

where

$$G^t = \begin{bmatrix} g_1 & g_2 & g_3 \end{bmatrix}$$

$$C^t = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \qquad 2.13$$

and

$$M = \begin{bmatrix} b_1 + b_4 - b_2 - b_3 & - b_2 & - b_3 \\ b_1 - b_4 + b_2 - b_3 & 2b_1 + b_2 & b_3 \\ b_3 + b_4 - b_1 - b_2 & b_2 & 2b_4 + b_3 \end{bmatrix}$$

2.14

In general, the differential equations for the elements of the network will be of the form of Eqs. 2.12 where the matrix M has elements which are linear combinations of the b_i and are arbitrary. Some of the b_i will be fixed by the requirement of invariance of a transfer function or the like. The remaining b_i are free and can be chosen to yield some optimal network. Note that given $B(x)$, Eqs. 2.12 are linear homogeneous differential equations, and this greatly simplifies the computational problem.

In the general RLC case, Eq. 2.12 would be modified by the addition of an equation for L.

3. Sensitivity Equations:

Consider the network function T(a driving point or transfer function). In general, T is a function of frequency and can be considered a function of the elements in the network. We are interested in determining how much the value of the transfer function T at a frequency changes when the value of one of the elements changes. To this end, define the sensitivity of the transfer function T to changes in the kth element

by the usual relation:[4]

$$S_k = \frac{e_k}{T} \frac{\partial T}{\partial e_k} \qquad 3.1$$

If now the given network is subjected to an equivalence transformation as described in the previous section, the elements vary continuously with x but the transfer function, being invariant, remains constant. As the elements change values, the sensitivity of T to changes in the elements also changes. Consider the derivative of sensitivity with respect to the independent variable x:

$$\frac{dS_k}{dx} = \sum_{i=1}^{b} \frac{\partial S_k}{\partial e_i} \frac{de_i}{dx} \qquad 3.2$$

where the summation is taken over all the elements of the network, and e_i denotes the ith element (R, L, or C).

By straightforward differentiation and noting that second partial derivatives can be taken in either order, it is easy to verify that

$$\frac{1}{e_k} \frac{\partial S_k}{\partial e_i} = \frac{1}{e_i} \frac{\partial S_i}{\partial e_k} \qquad 3.3$$

Using Eq. 3.3 in Eq. 3.2 yields

$$\frac{dS_k}{dx} = \sum_{i=1}^{b} \frac{e_k}{e_i} \frac{\partial S_i}{\partial e_k} \frac{de_i}{dx} \qquad 3.4$$

But the transfer function T is a function of the elements e_i and is independent of x. Hence

$$\frac{dT}{dx} = \sum_{i=1}^{b} \frac{\partial T}{\partial e_i} \frac{de_i}{dx} = 0 \qquad\qquad 3.5$$

If Eq. 3.1 is rearranged as

$$\frac{\partial T}{\partial e_i} = \frac{TS_i}{e_i} \qquad\qquad 3.6$$

Eq. 3.5 yields

$$T \sum_{i=1}^{b} \frac{S_i}{e_i} \frac{de_i}{dx} = 0 \qquad\qquad 3.7$$

Dividing Eq. 3.7 by T and taking the partial derivative with respect to e_k yields

$$\sum_{i=1}^{b} \left\{ \frac{e_i'}{e_i} \frac{\partial S_i}{\partial e_k} + \frac{S_i}{e_i} \frac{\partial e_i'}{\partial e_k} - \frac{e_i' S_i}{e_i^2} \frac{\partial e_i}{\partial e_k} \right\} \qquad 3.8$$

Rearranging Eq. 3.8 supplies the factor needed for Eq. 3.4 and noting that $\partial e_i / \partial e_k$ is zero for i $\neq$ k and unity for i = k, yields

$$\sum_{i=1}^{b} \frac{1}{e_i} \frac{\partial S_i}{\partial e_k} \frac{de_i}{dx} = \sum_{i=1}^{b} \left\{ -\frac{S_i}{e_i} \frac{\partial e_i'}{\partial e_k} \right\} + \frac{S_k}{e_k^2} \frac{de_k}{dx}$$

$$3.9$$

Substituting Eq. 3.9 into Eq. 3.4 yields a differential equation for the sensitivity:

$$\frac{dS_k}{dx} = \frac{S_k}{e_k} \frac{de_k}{dx} \sum_{i=1}^{b} \frac{e_k S_i}{e_i} \frac{\partial}{\partial e_k} \left(\frac{de_i}{dx} \right) \qquad 3.10$$

This equation simplifies considerably if an additional variable q_k, related to S_k by

$$S_k = q_k e_k \qquad 3.11$$

is defined.

Then

$$\frac{dS_k}{dx} = q_k \frac{de_k}{dx} + e_k \frac{dq_k}{dx} \qquad 3.12$$

With this change of variable, Eq. 3.10 simplifies to

$$\frac{dq_k}{dx} = - \sum_{i=1}^{b} q_i \frac{\partial}{\partial e_k} \left(\frac{de_i}{dx} \right) \qquad 3.13$$

In section 2, the differential equations for the elements were shown to be of the form

$$\frac{dG}{dx} = MG$$

$$\frac{dC}{dx} = MC$$

$$\frac{dL}{dx} = ML \qquad 3.14$$

Hence in Eq. 3.13, if e_i is a conductance for example, $\dfrac{de_i}{dx}$ is

a function only of the conductances---there is no coupling

in Eqs. 3.14 between the various element-kinds. Hence unless

e_k is also a conductance, $\dfrac{\partial}{\partial e_k} \left(\dfrac{de_i}{dx} \right)$ will be zero. That is,

in Eq. 3.13, if e_k is a conductance, only those values of i

corresponding to the conductances of the network appear and if

e_k is a capacitance, only those values of i corresponding to

capacitances in the network will appear, etc. Hence the dif-

ferential equations for the sensitivities will be of a form

similar to Eqs. 3.14 above, in that the equations for one kind

of element will not involve the other element-kinds.

To display these equations succinctly, define S_{Gk}, S_{Ck},

S_{Lk} the sensitivity of the kth conductance, kth capacitance,

and kth inductance respectively and similarly for q_{Gk}, q_{Ck},

q_{Lk}. Then Eq. 3.13 becomes

$$\frac{dq_{Gk}}{dx} = - \sum_i q_{Gi} \frac{\partial}{\partial g_k} \left(\frac{dg_i}{dx} \right)$$

$$\frac{dq_{Ck}}{dx} = - \sum_i q_{Ci} \frac{\partial}{\partial C_k} \left(\frac{dC_i}{dx} \right)$$

$$\frac{dq_{Lk}}{dx} = - \sum_i q_{Li} \frac{\partial}{\partial L_k} \left(\frac{dL_i}{dx} \right) \qquad 3.15$$

From Eq. 3.14, it is clear that

$$\frac{dq_i}{dx} = u_i^t M G \qquad 3.16$$

and

$$\frac{\partial}{\partial g_k} \left(\frac{dg_i}{dx} \right) = u_i^t M G u_j \qquad 3.17$$

where u_r is the rth unit vector. Hence the sensitivity equations for the conductances (the others are similar) simplify as follows:

$$\frac{dq_{Gk}}{dx} = - \sum_i q_{Gi} u_i^t M u_j \qquad 3.18$$

$$= - u_j^t M^t Q_G \qquad 3.19$$

where

$$Q_G^t = \left[q_{G1} q_{G2} \cdots \right] \qquad 3.20$$

The equations for all of the sensitivities in matrix form are then simply

$$\frac{dQ_G}{dx} = - M^t Q_G \qquad 3.21$$

The equations for the capacitance and inverse inductance sensitivities are identical in form:

$$\frac{dQ_C}{dx} = - M^t Q_C$$

$$\frac{dQ_L}{dx} = - M^t Q_L \qquad\qquad 3.22$$

Thus, the equations for the sensitivities of the transmission are merely the adjoint equations. The actual sensitivities are then determined by Eq. 3.11. To summarize, six sets of first order linear homogeneous differential equations for the three element kinds (R, L, and C) and the sensitivities of the transfer function of the network have been found such that their solution--for each value of x--gives a set of elements of an equivalent network, one having the same transfer function-- and also the sensitivity of that network to changes in each of its elements. The given network and its sensitivities serve as initial conditions for these differential equations. The particular transfer function being held invariant is deter- mined by the rows of the matrix B which are set equal to zero. The matrix M is derived from B by the method discussed in sec- tion 2. It is easy to formalize the whole procedure by adopt- ing standard numbering schemes etc. in order to give formulae for M in terms of B, but this adds nothing conceptually or even practically and therefore no space is devoted to this problem here.

There remains the problem of determining the network equivalent to the given network with minimum sensitivity. This is done in a straightforward manner by first defining a criterion.

At any frequency, sensitivity may be complex, but only the magnitude is of interest. Choose a criterion which is essentially the sum of the magnitudes-squared of the sensitivities of interest:

$$\phi = \frac{1}{2} \sum_k \left[q_{Gk} q_{Gk}^* + q_{Ck}^* + q_{Lk} q_{Lk}^* \right] \qquad 3.23$$

where * indicates conjugate and where the summation is taken over those k's of interest.

The method of steepest descent is most useful for minimizing $\dot{\phi}$. Using Eq. 3.21 and 3.22, the derivative of ϕ may be written as

$$\frac{d\phi}{dx} = - Q_G^t M_s Q_G^* - Q_C^t M_s Q_C^* - Q_L^t M_s Q_L^* \qquad 3.24$$

Here M_s is the symmetric part of M and the fact that M is real is used to derive Eq. 3.24. Eq. 3.24 is pure real as desired.

For steepest descent to the minimum, $d\phi/dx$ should be as negative as possible. Thus the free b_i which appear in M should be chosen to make this so. But multiplying out Eq. 3.24 shows that $d\phi/dx$ is linear in the b_i and since the elements of B were limited in magnitude to unity, $d\phi/dx$ is minimized if all of the b's take on plus or minus one, depending

upon the sign of its coefficient in Eq. 3.24. In the next section, it is shown that this set of differential equations is extremely easy to solve on a digital computer. In this case, at each step of the solution, the machine merely chooses the proper b's to make the descent toward the minimum as swift as possible. The only constraint on the choice is that no element go negative at any step. Since at each step the elements change only slightly (in the numerical integration of a differential equation, small increments are always chosen), it is easy to modify the computation so that the elements stay positive (because the changes in any single element are linearly related to the b's by Eqs. 2.12).

The sensitivity equations give solutions at only a single frequency and hence if it is desired to determine sensitivities at more than one frequency, the equations must be solved at each frequency. This does not appreciably complicate the computing because the only change in the differential equations is in the initial conditions and most of the computation involves the matrix of coefficients M or $- M^t$ which is independent of frequency.

4. Computer Synthesis:

It has been shown in the previous sections that equivalent networks may be generated from the solution of a set of linear first order differential equations. In this section,

an efficient computational procedure for the integration of such equations, leading to efficient machine synthesis of minimal sensitive networks is discussed.

Consider the vector differential equation

$$\frac{dV}{dx} = AV \qquad\qquad 4.1$$

where V is an n x 1 matrix and A is an n x n matrix. It is well known that the solution may be written in terms of a matrix exponential:[12]

$$V(x) = e^{Ax} V(0) \qquad\qquad 4.2$$

The matrix exponential is defined by the (absolutely and uniformly convergent) series[12]

$$e^{Ax} = u + Ax + \frac{1}{2} A^2 x^2 + \frac{1}{6} A^3 x^3 + \dots + \frac{1}{k!} A^k x^k + \dots \qquad\qquad 4.3$$

For machine solution, we are interested in solutions at $x = 0$, Δx, $2\Delta x$, $3\Delta x$, $4\Delta x$, ... where Δx is small. In this case, the series above may be approximated by its first few terms because x is small. Thus the computer must form A, A^2, and A^3 if four terms are used and then form the weighted sum. Such a calculation is well suited to computers, making the integra-

tion process straightforward.

In the case of the problem at hand, the one set of equations has matrix M and the other $- M^t$. Hence the matrix exponentials which must be calculated are

$$e^{Mx} \quad \text{and} \quad e^{-M^t x}$$

From Eq. 4.3, it is clear that the same matrices, M, M^2, and M^3 can be used for both calculations, the only difference in the two computations being that the terms are added with proper signs.

The computation proceeds with the following steps. First, the values of the elements and sensitivities are used in Eq. 3.24 to determine the optimum b's for the next step. Second, the matrices M, M^2, and M^3 are formed. Third, e^{Mx} and $e^{-M^t x}$ are formed. Fourth, the new elements and sensitivities are calculated by multiplying the matrix exponentials by the present element values and sensitivities as in Eq. 4.2. Fifth, the elements are checked to determine if any has gone negative. If so, the step is repeated with the choice of b's modified so that the element which went negative remains zero or positive. Then the procedure is repeated until a minimum is reached.

Although derived for RLC networks, the theory of equivalent networks is also valid for active networks with only

slight modifications. As an example of this procedure, a network containing two vacuum tubes and 30 RLC elements was chosen. The parameters of the tubes were fixed and it was desired to generate equivalent networks with the voltage transfer function held invariant so that the voltage transfer function was minimally sensitive to the gains of the tubes. Because the tube parameters were assumed fixed, the RLC formulation derived in this paper is applicable.

Four discrete frequencies were chosen and the linear combination of the sensitivities at these frequencies minimized. Using a Burroughs 220 computer, several minutes of computing time led to a 25% decrease in sensitivity of this network, and required only 6 steps. No attempt was made to make the program optimal and a more efficient program should decrease this time still further. In any case, the computing time for even complex networks does not limit the application of this approach.

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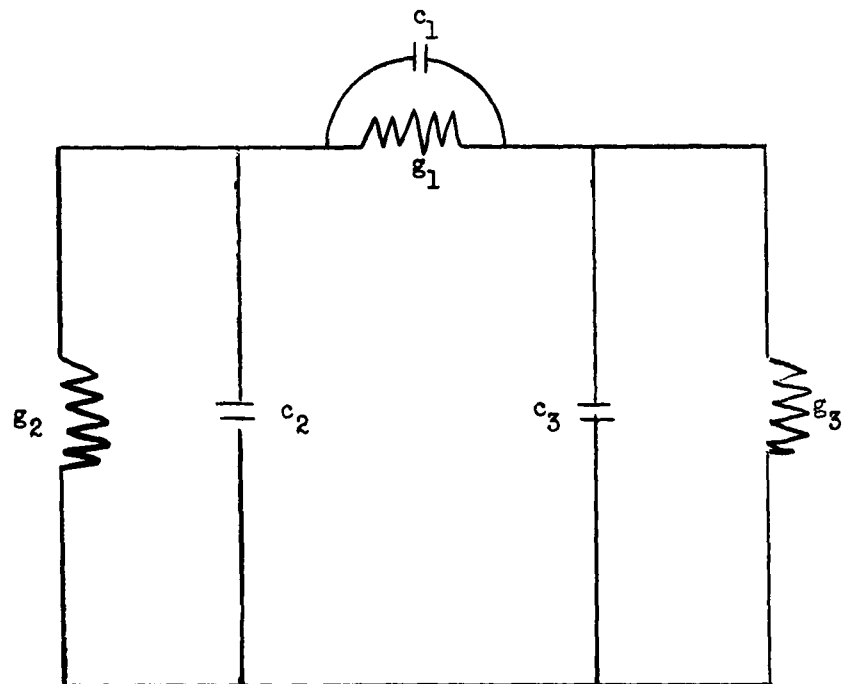
Fig. 1

Example RC Network

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Dr. Nathan Marcuvitz
Polytechnic Inst. of Brklyn.
55 Johnson Street
Brooklyn 1, New York

Commanding Officer
ONR Branch Office
346 Broadway
New York 13, N. Y.

National Security Agency
Division of Physical Sciences
Ft. George G. Meade,
Maryland

Commanding Officer
ONR Branch Office
495 Summer Street
Boston 10, Massachusetts